WEIGHTING THE CLUSTERS OF GALAXIES WITH WEAK GRAVITATIONAL LENSING: The problem of the mass-sheet degeneracy

M. Bradač[1,2], M. Lombardi[1,3], P. Schneider[1]
[1]*Institut für Astrophysik und Extraterrestrische Forschung, Auf dem Hügel 71, D-53121 Bonn, Germany* [2]*Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany*
[3]*European Southern Observatory, Karl-Schwarzschild-Str. 2, D-85748 Garching bei München, Germany*

Weak gravitational lensing is considered to be one of the most powerful tools to study the mass and the mass distribution of galaxy clusters. However, weak lensing mass reconstructions are plagued by the so-called mass-sheet degeneracy – the surface mass density κ of the cluster can be determined only up to a degeneracy transformation $\kappa \to \kappa' = \lambda\kappa + (1-\lambda)$, where λ is an arbitrary constant. This transformation fundamentally limits the accuracy of cluster mass determinations if no further assumptions are made. We discuss here a possibility to break the mass-sheet degeneracy in weak lensing mass maps using distortion and redshift information of background galaxies. Compared to other techniques proposed in the past, it does not rely on any assumptions on cluster potential and does not make use of weakly constrained information (such as the source number counts, used in the magnification effect). Our simulations show that *we are effectively able to break the mass-sheet degeneracy for supercritical lenses* and that for undercritical lenses the mass-sheet degeneracy is very difficult to be broken, even under idealised conditions.

1 Introduction

Weak gravitational lensing techniques have been to great extent applied to measure the cluster mass distribution. Unfortunately, all these methods suffer from the fact that the projected surface mass density κ can be determined only up to a degeneracy transformation $\kappa \to \kappa' = \lambda\kappa+(1-\lambda)$, where λ is an arbitrary constant. This invariance fundamentally limits the accuracy of cluster mass determinations if no further assumptions are made. In particular, as we will show later this transformation leaves the main observable unchanged and therefore λ can not be directly constrained.

A naive solution to the problem of mass-sheet degeneracy is to constrain λ by making simple assumptions about κ. For example, one can assume that the surface mass density is decreasing

with distance from the centre, implying $\lambda > 0$. In addition, κ is likely to be non-negative, and so one can obtain an upper limit on λ (for $\kappa < 1$).

More quantitatively, with the use of wide field cameras one might try to assume that $\kappa \simeq 0$ at the boundary of the field, far away from the cluster center. However, if we consider for example a $M_{\rm vir} = 10^{15} M_\odot$ cluster at redshift $z = 0.2$, we expect from N-body simulations to have a projected dimensionless density of about $\kappa \simeq 0.005$ at 15 arcmin from the cluster center (Douglas Clowe, private communication). Hence, even with the use of a 30×30 arcmin camera we expect to underestimate the virial mass of such a cluster by $\sim 20\%$. Therefore additional constraints need to be used. We show in these proceedings (more details can be found in [1]), that background (photometric) redshifts can help us to lift this degeneracy and therefore remove the fundamental limit on cluster mass reconstructions.

2 Principles of weak gravitational lensing

Weak gravitational lensing measures the strength of the gravitational field from a sample of measured ellipticities of background galaxy images. Under the assumption that the intrinsic ellipticity distribution is isotropic, $\langle \epsilon^{\rm s} \rangle = 0$, the expectation value for the lensed, image ellipticities at redshift z becomes

$$\langle \epsilon(z) \rangle = \begin{cases} g(\boldsymbol{\theta}, z) & \text{if } \left| g(\boldsymbol{\theta}, z) \right| < 1 \,, \\ 1/g^*(\boldsymbol{\theta}, z) & \text{otherwise} \,. \end{cases} \tag{1}$$

The redshift-dependent reduced shear $g(\boldsymbol{\theta}, z)$ is given by

$$g(\boldsymbol{\theta}, z) = \frac{Z(z)\gamma(\boldsymbol{\theta})}{1 - Z(z)\kappa(\boldsymbol{\theta})} \,, \tag{2}$$

where $Z(z)$ is the so-called "cosmological weight" function [2]. By measuring an ensemble average of the lensed image ellipticities, an unbiased estimator for the reduced shear can be obtained. The $Z(z)$ function accounts for the fact that the sources that have a redshift z smaller than the deflector $z_{\rm d}$ are not lensed ($Z(z \leq z_{\rm d}) = 0$) and asymptotically increases to 1 for $z \to \infty$. Note that, as suggested by its name, $Z(z)$ is cosmology dependent. In [2] the authors have shown that the differences between Einstein-de Sitter and the nowadays assumed standard cosmology ($\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.7$) are not significant for the purpose of cluster-mass reconstructions. Therefore we will from now on use Einstein-de Sitter cosmology.

2.1 The problem of the mass sheet degeneracy

In the simple case of background sources all having the *same redshift*, the mass-sheet degeneracy can be understood just using the above equations. Indeed, consider for a moment the transformation of the potential ψ

$$\psi(\boldsymbol{\theta}, z) \to \psi'(\boldsymbol{\theta}, z) = 0.5\ (1 - \lambda)\, \boldsymbol{\theta}^2 + \lambda \psi(\boldsymbol{\theta}, z) \,, \tag{3}$$

where λ is an arbitrary constant. κ and γ are related to the potential ψ through its second partial derivatives (denoted by subscript), namely $\kappa = 0.5\,(\psi_{,11} + \psi_{,22})$, $\gamma_1 = 0.5\,(\psi_{,11} - \psi_{,22})$, $\gamma_2 = \psi_{,12}$. From (3) it follows that κ transforms as

$$\kappa(\boldsymbol{\theta}, z) \to \kappa'(\boldsymbol{\theta}, z) = \lambda \kappa(\boldsymbol{\theta}, z) + (1 - \lambda) \,, \tag{4}$$

and the shear $\gamma(\boldsymbol{\theta}, z)$ changes to $\lambda\gamma(\boldsymbol{\theta}, z)$. Therefore the reduced shear $g(\boldsymbol{\theta}, z)$ (our main observable) remains invariant.

The authors in [3] have shown that in the case of a known *redshift distribution*, a similar form of the mass-sheet degeneracy holds to a very good approximation for non-critical clusters (i.e.

for clusters with $|g(\boldsymbol{\theta}, z)| \leq 1$ for all source redshifts). In such a case the standard weak-lensing mass reconstruction is affected by the degeneracy

$$\kappa \to \kappa' \simeq \lambda\kappa + (1 - \lambda)\big\langle Z(z)\big\rangle / \big\langle Z^2(z)\big\rangle \,, \tag{5}$$

where $\big\langle Z^n(z)\big\rangle$ denotes the n-th order moment of the distribution of cosmological weights. As a result, *standard* weak-lensing reconstructions are still affected by the mass-sheet degeneracy even for sources at different redshifts; moreover, simulations show that the degeneracy is hardly broken even for the lenses close to critical.

In this work we use the information of *individual redshifts* of background sources for to break this degeneracy. Indeed, suppose for simplicity that half of the background sources are located at a known redshift $z^{(1)}$, and the other half at another known redshift $z^{(2)}$. Then, the weak lensing reconstructions based of the two populations will provide two different mass maps, $\kappa'(\boldsymbol{\theta}, z^{(1)})$ and $\kappa'(\boldsymbol{\theta}, z^{(2)})$, leading to two different forms of the mass-sheet degeneracy. In other words, the two mass reconstructions ($i = 1, 2$) are given by

$$\kappa'(\boldsymbol{\theta}, z^{(i)}) = \lambda^{(i)} \kappa_{\mathrm{t}}(\boldsymbol{\theta}, z^{(i)}) + \big(1 - \lambda^{(i)}\big) \tag{6}$$

where we have denoted $\kappa_{\mathrm{t}}(\boldsymbol{\theta}, z^{(i)})$ the true projected κ of the lens at the angular position $\boldsymbol{\theta}$ for sources at redshift $z^{(i)}$. Since the transformation (6) holds for any $\boldsymbol{\theta}$, we have a system of equations to be solved for $\lambda^{(1)}$ and $\lambda^{(2)}$. The relation between $\kappa_{\mathrm{t}}(\boldsymbol{\theta}, z^{(1)})$ and $\kappa_{\mathrm{t}}(\boldsymbol{\theta}, z^{(2)})$ is known, namely $\kappa_{\mathrm{t}}(\boldsymbol{\theta}, z^{(1)}) Z(z^{(2)}) = \kappa_{\mathrm{t}}(\boldsymbol{\theta}, z^{(2)}) Z(z^{(1)})$. Suppose one measures both $\kappa'(\boldsymbol{\theta}, z^{(i)})$ at N different positions $\boldsymbol{\theta}_j$, this gives us a system of $2N$ equations to be solved for $\lambda^{(i)}$ and $\kappa_{\mathrm{t}}(\boldsymbol{\theta}_j)$. This theoretically allows us to break the mass-sheet degeneracy.

It is interesting to observe that this argument only applies to relatively "strong" lenses. Indeed, for "weak" lenses, i.e. lenses for which we can use a first order approximation in κ and γ, the expectation value of measured image ellipticities is $\big\langle \epsilon(z) \big\rangle = \gamma(\boldsymbol{\theta}, z)$. In such case the degeneracy of the form $\psi(\boldsymbol{\theta}, z) \to \psi'(\boldsymbol{\theta}, z) = 0.5\,(1 - \lambda)\,\boldsymbol{\theta}^2 + \psi(\boldsymbol{\theta}, z)$ leaves the observable $\gamma(\boldsymbol{\theta}, z)$ unchanged. As a result, the method described above cannot be used to break the mass-sheet degeneracy for these lenses. Only when the $(1 - Z(z)\kappa)$ term in the reduced shear becomes important and $g(\boldsymbol{\theta}, z)$ can be distinguished from $\gamma(\boldsymbol{\theta}, z)$ in the (noisy) data, we will be able to make unbiased cluster mass reconstructions.

3 Results

In order to test whether we can break the mass-sheet degeneracy by using redshift information on the background sources, we performed a simple test on simulated data.

We assume parametrised model families for the underlying cluster mass distribution and use the likelihood analysis (described in detail in [1]). We consider a non-singular model that approximates an isothermal sphere for large distances in which we allow for a constant sheet in surface mass density. The dimensionless surface mass density is given by

$$\kappa(\theta/\theta_{\mathrm{c}}) = \kappa_0 \left(1 + \theta^2 / \left(2\theta_{\mathrm{c}}^2\right)\right) \left(1 + \theta^2/\theta_{\mathrm{c}}^2\right)^{-3/2} + \kappa_1 \,, \tag{7}$$

where κ_0, κ_1 are dimensionless constants and θ_{c} is the core radius.

Figure 1 shows the results of log-likelihood minimisation of four different lens parameters, for each of them 100 mock realisations were calculated. Solid and dashed lines in the figure correspond to the expected mass-sheet degeneracy calculated using Eq. (5). We use different weighting schemes to calculate $\big\langle Z^n(z)\big\rangle$. A best fit to the data is given by weighting each galaxy with the inverse of σ_i^2

$$\sigma_i^2 = \left(1 - \big|\langle\epsilon\rangle\big|^2\right)^2 \sigma_{\epsilon^{\mathrm{s}}}^2 + \sigma_{\mathrm{err}}^2 \,, \tag{8}$$

where σ_i is an approximation for the true dispersion of measured ellipticities (see caption of Fig. 1 for detailed description).

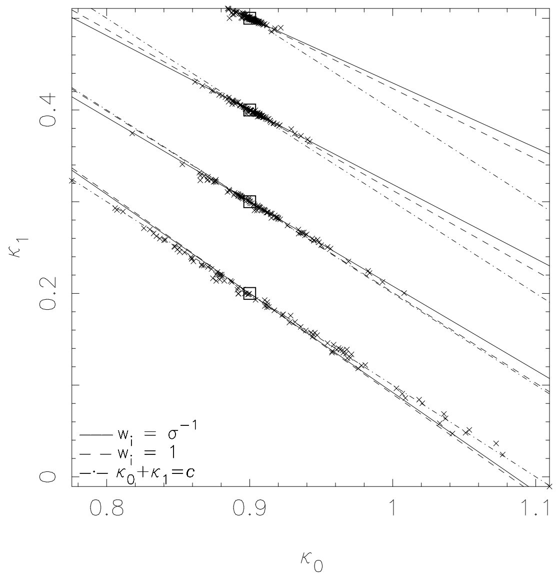

Figure 1: Recovered parameter values (crosses) as a result of minimising the log-likelihood function. For each of the four sets of parameters (denoted by squares) 100 mock catalogues were created. For these $N_{\rm g} = 2000$ galaxies were drawn randomly across the field of 6×6 arcmin^2. The intrinsic ellipticities $\epsilon^{\rm s}$ were drawn from a Gaussian distribution characterised by $\sigma_{\epsilon^{\rm s}} = 0.15$. We draw the redshifts of the background sources following a Γ-distribution with $z_0 = 2/3$. We put the lens at a redshift $z_{\rm d} = 0.2$. The measurement errors $\epsilon^{\rm err}$ on the observed ellipticities were drawn from a Gaussian distribution with $\sigma_{\rm err} = 0.1$ and added to the lensed ellipticities, for the redshift errors we use $\sigma_{\rm zerr} = 0.06\,(1 + z_i)$. Solid and dashed lines correspond to the expected mass-sheet degeneracy calculated using $w_i = 1/\sigma_i^2$ and $w_i =$ const. for the weighting scheme respectively (both almost overlap). Dot-dashed lines are given by $\kappa_0 + \kappa_1 =$ const.

4 Conclusions

We considered a new method to break the mass-sheet degeneracy in weak lensing mass reconstructions using shape measurements only. Our main conclusions are:
(1) The mass-sheet degeneracy can be effectively broken by using redshift information of the individual sources. However, this is effective for critical clusters only, i.e. for clusters that have sizable regions where multiple imaging is possible (and thus perhaps observed). The statistical lensing analysis has to be employed close to and inside the critical curves of the cluster. In the regions far outside the critical curve, where weak lensing mass reconstructions are normally performed, the lens is too weak for the mass-sheet degeneracy to be broken by using redshift and distortion information only, even when idealised conditions are employed.
(2) Using simulations we find that correlations remaining from the mass-sheet degeneracy transformation for critical lenses are well described by $\kappa \to \kappa' \simeq (1-\lambda)\kappa + (1-\lambda)\big\langle Z(z)\big\rangle / \big\langle Z^2(z)\big\rangle$, where the moments of the cosmological weights are calculated using (8).
(3) In order to break the mass-sheet degeneracy with current data it is necessary to extend the statistical lensing analysis closer to the cluster centre and to simultaneously perform weak and strong lensing analysis of the cluster. This will be a subject of a future study.

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References

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